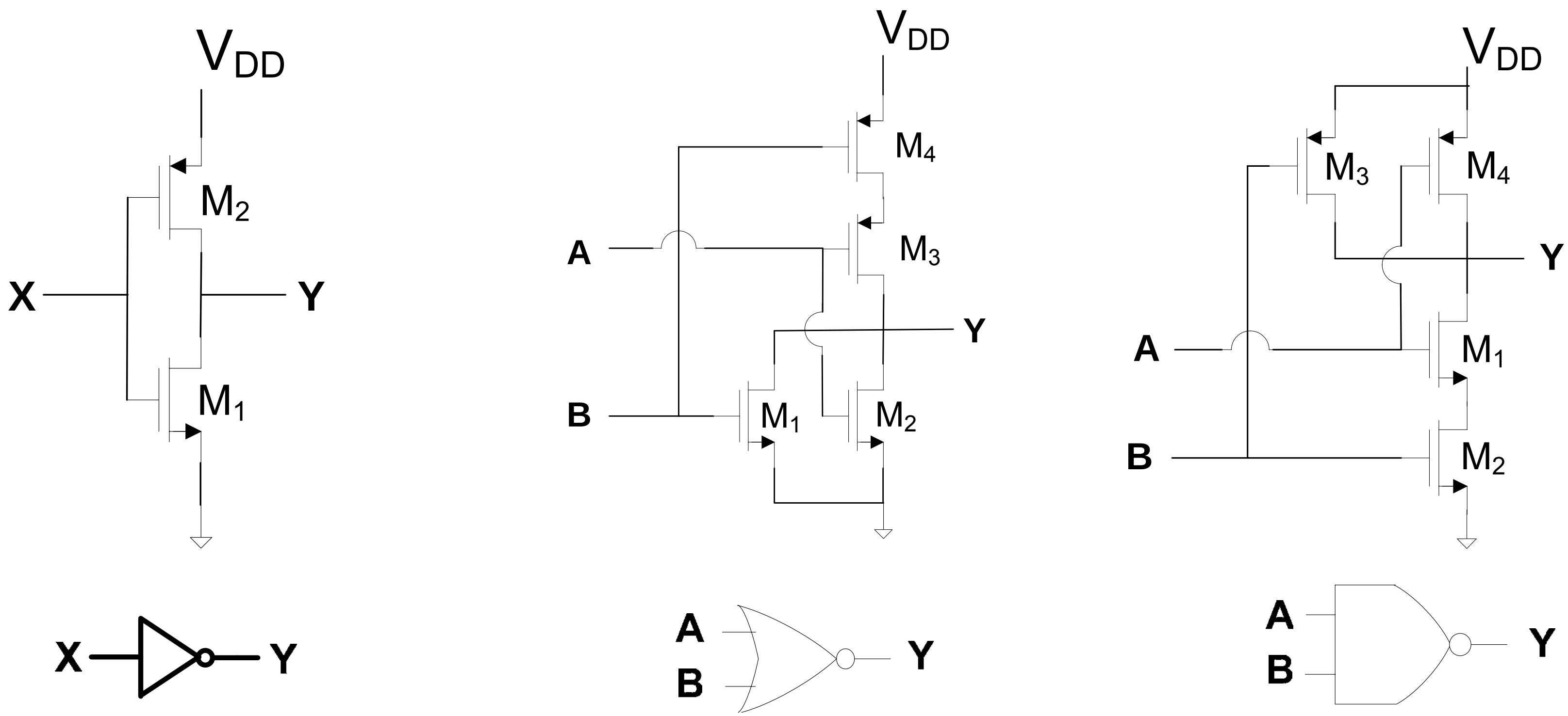
EE 230
Lecture 34

Small Signal Models

Small Signal Analysis

Review from Last Time:

MOS Transistor Applications (Digital Circuits)

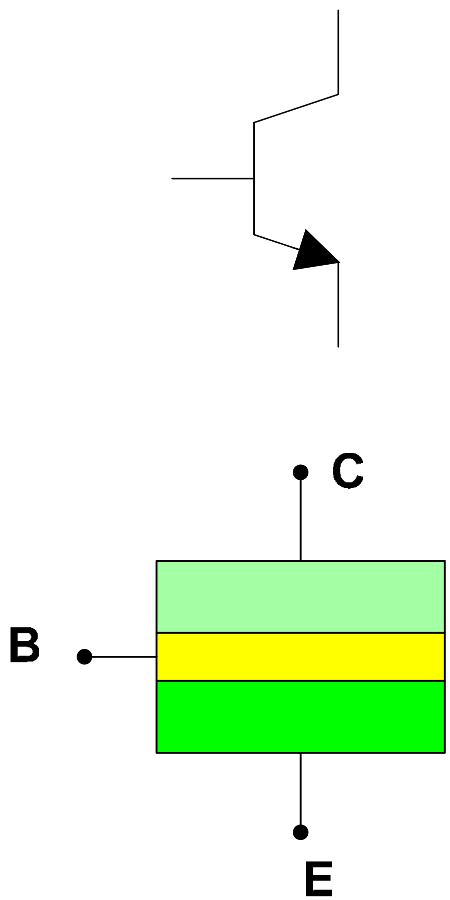


- Termed CMOS Logic
- Widely used in industry today (millions of transistors in many ICs using this logic
- Almost never used as discrete devices

Review from Last Time:

Bipolar Transistor

npn



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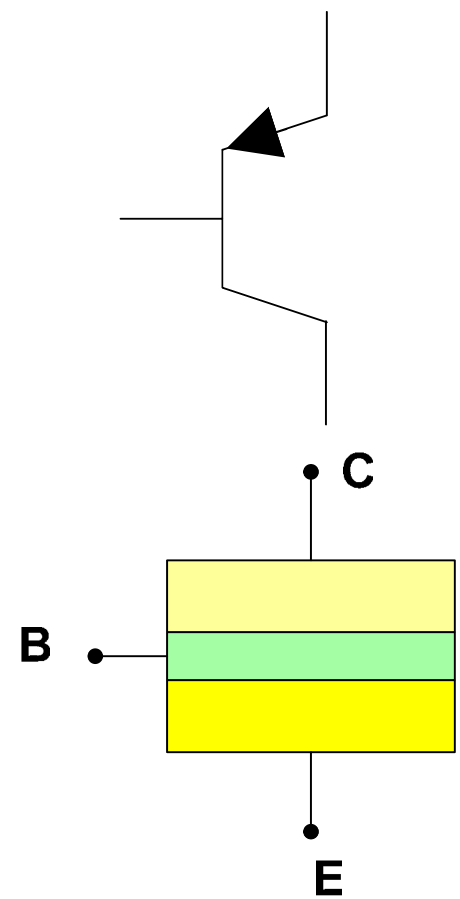
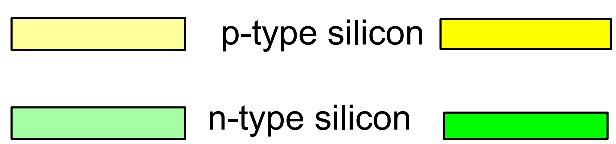
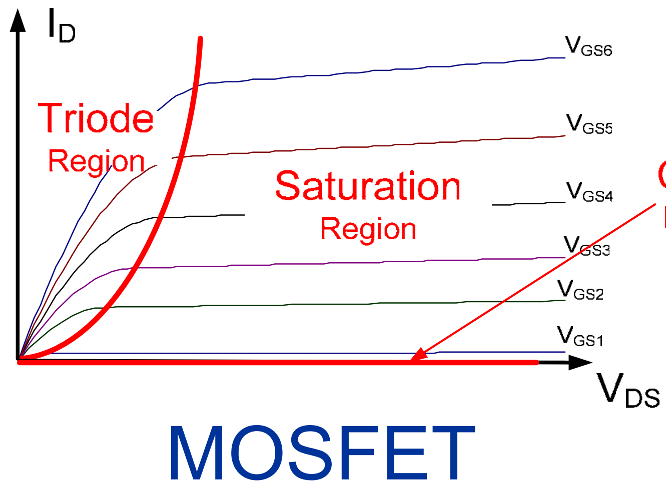
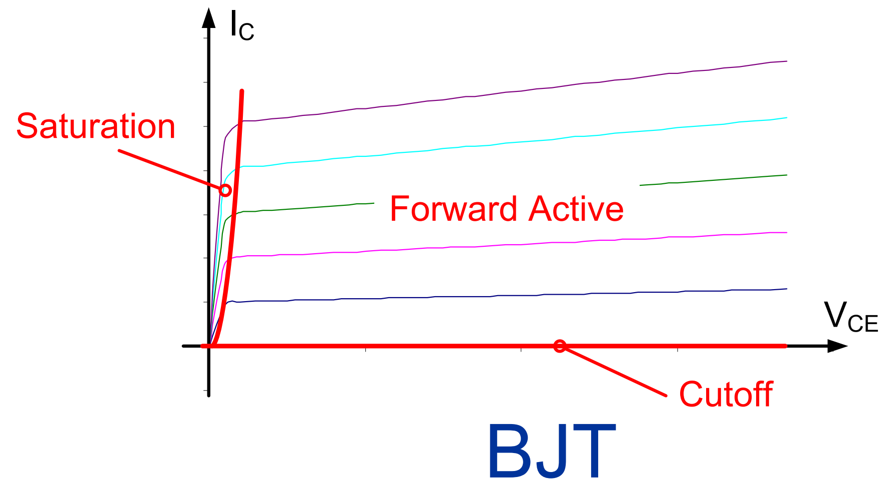




Review from Last Time:

Bipolar and MOS Region Comparisons





Cutoff	⟷	Cutoff
Saturation	⟷	Forward Active
Triode	⟷	Saturation

Review from Last Time:

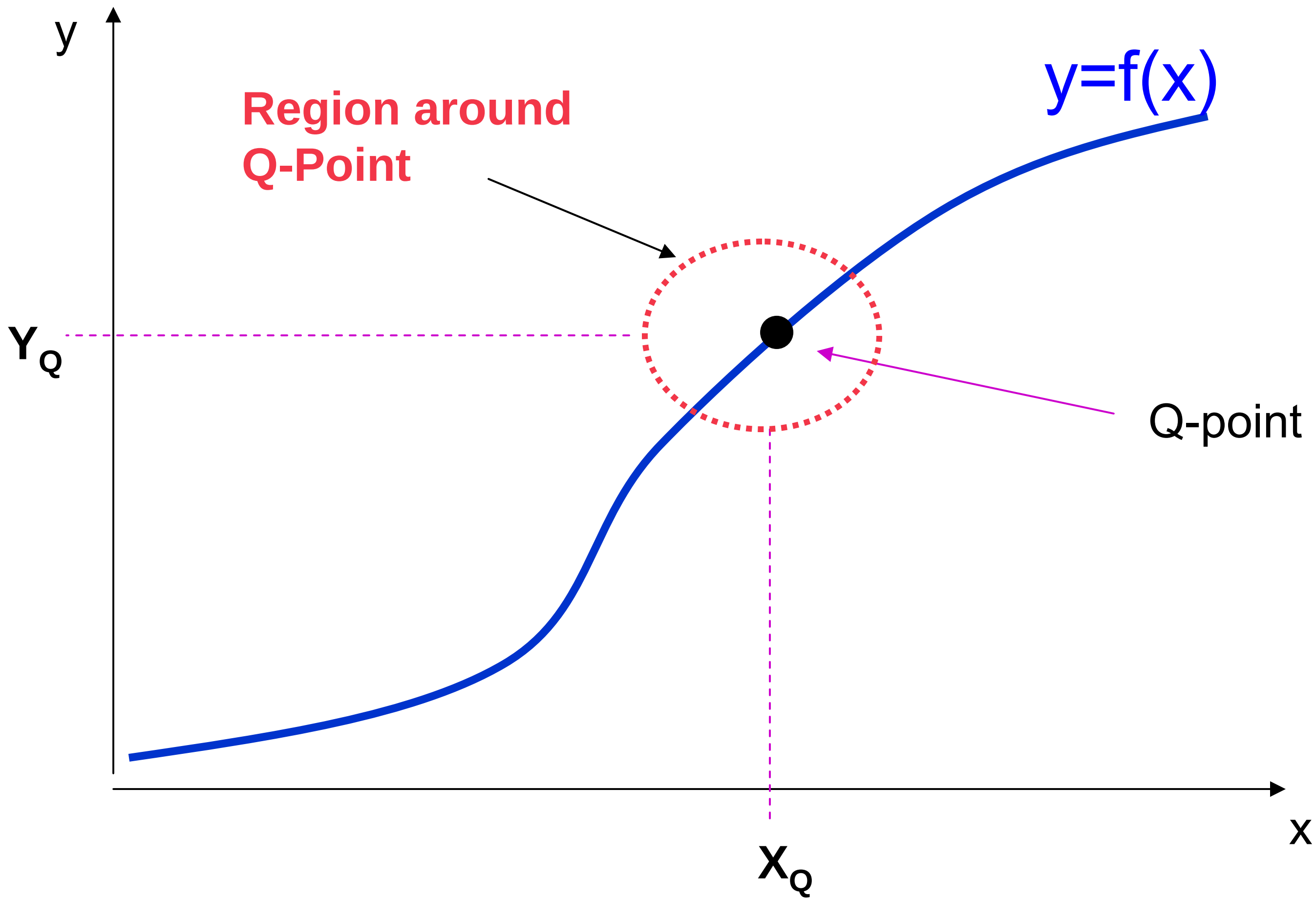
Bipolar Transistor

Multi-Region Model

$$I_C = \beta I_B \left(1 + \frac{V_{CE}}{V_{AF}}\right)$$ $$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$ $$V_t = \frac{kT}{q}$$	V_{BE}>0.4V V_{BC}<0	Forward Active
V_{BE}=0.7V V_{CE}=0.2V	$I_C<\beta I_B$	Saturation
$I_C=I_B=0$	V_{BE}<0 V_{BC}<0	Cutoff

Review from Last Time:

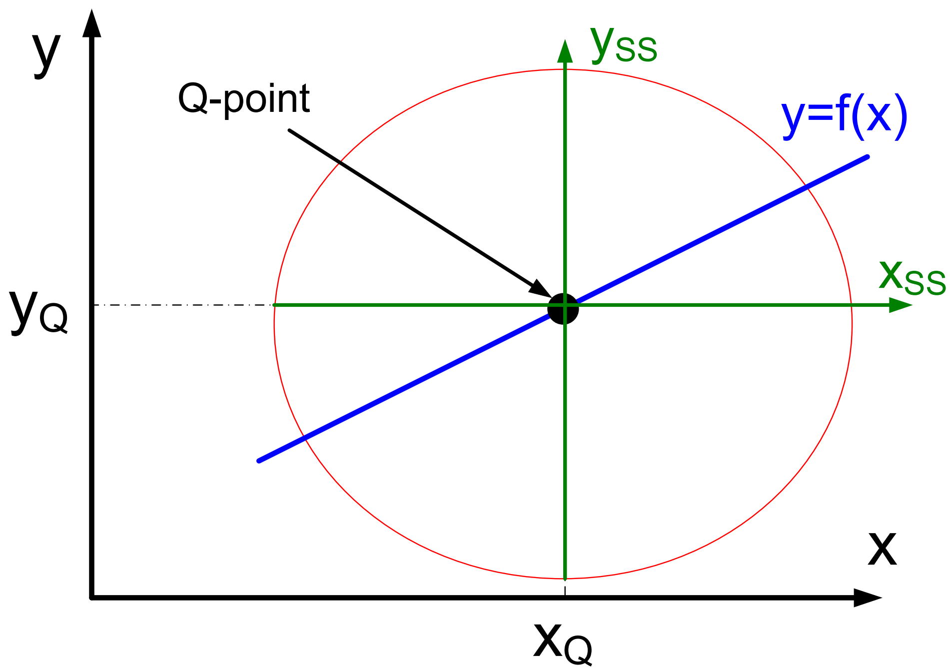
Small-Signal Principle



Relationship is nearly linear in a small enough region around Q-point
Region of linearity is often quite large
Linear relationship may be different for different Q-points

Review from Last Time:

Small-Signal Principle

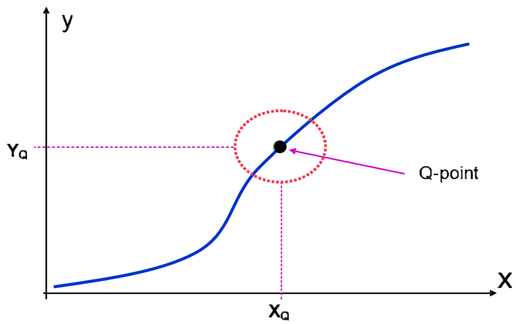


Small-Signal Model:

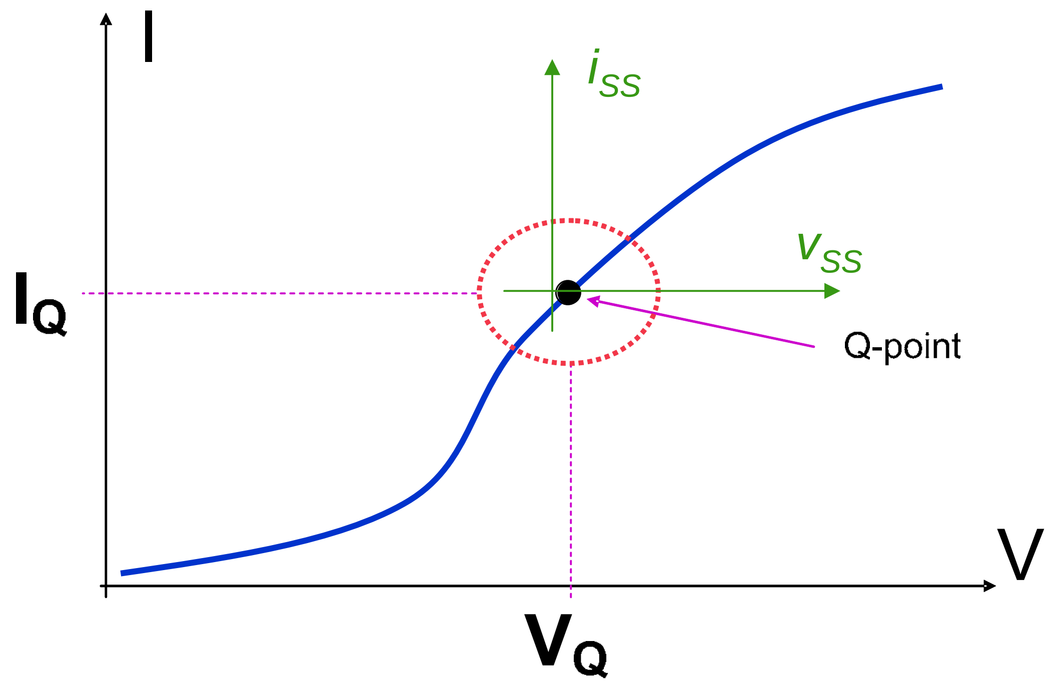
$$y = f(x_Q) + \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} (x - x_Q) \quad \Longleftrightarrow \quad y_{ss} = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} x_{ss}$$

- *Mathematically, small signal model is simply Taylor's series expansion at the Q-point truncated after first-order terms*

Small-Signal Principle

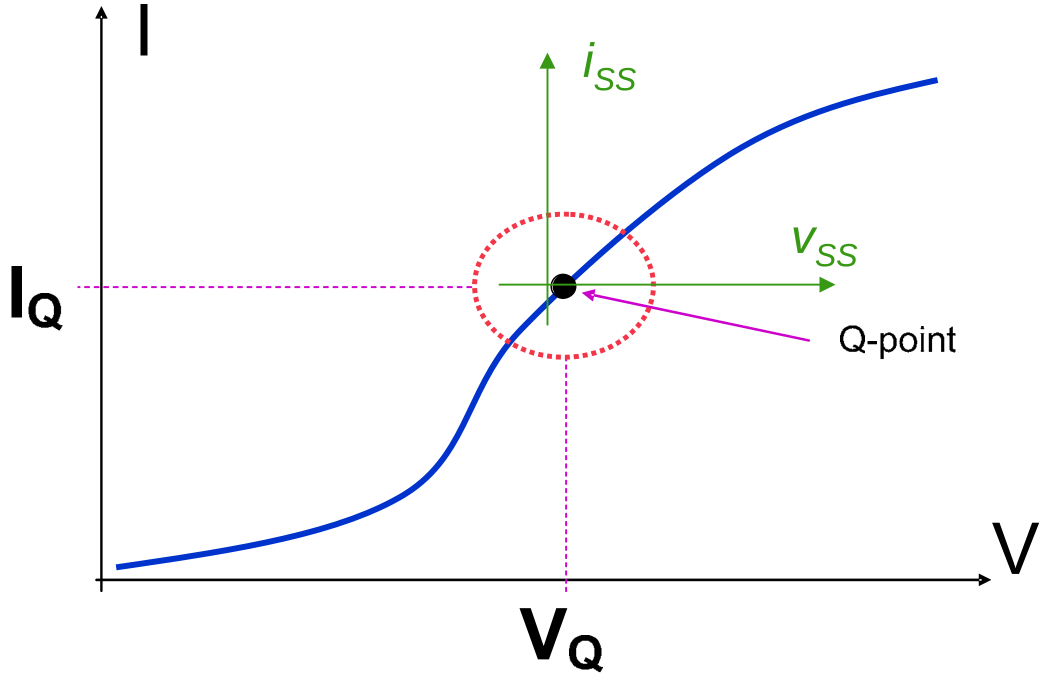


$$y_{ss} = \left.\frac{\partial f}{\partial x}\right|_{x=x_Q} x_{ss}$$



$$i_{ss} = \left.\frac{\partial I}{\partial V}\right|_{V=V_Q} v_{ss}$$

Small-Signal Principle



$$i_{ss} = \left.\frac{\partial I}{\partial V}\right|_{V=V_Q} v_{ss}$$

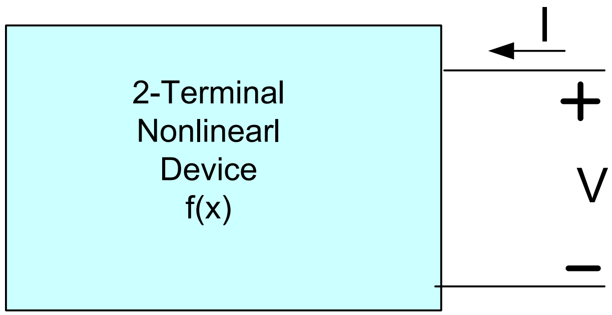
$$i_{ss} \overset{\text{def}}{=} \boldsymbol{i}$$

$$v_{ss} \overset{\text{def}}{=} \boldsymbol{v}$$

$$y \overset{defn}{=} \left.\frac{\partial I}{\partial V}\right|_{V=V_Q}$$

$$\boxed{\boldsymbol{i} = y\,\boldsymbol{v}}$$

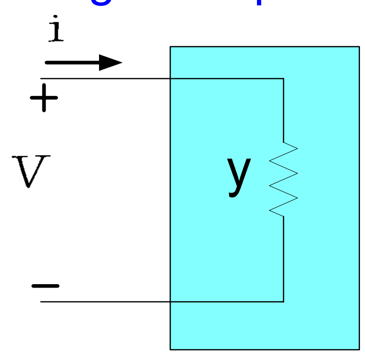
Small-Signal Principle



$$y = \left.\frac{\partial I}{\partial V}\right|_{V=V_Q}$$

$$\boldsymbol{i} = y\,\boldsymbol{v}$$

A Small Signal Equivalent Circuit



The small-signal model of this 2-terminal electrical network is a resistor of value 1/y
One small-signal parameter characterizes this one-port but it is dependent on Q-point

Review from Last Time:

Small-Signal Principle

Goal with small signal model is to predict performance of circuit or device in the vicinity of an operating point

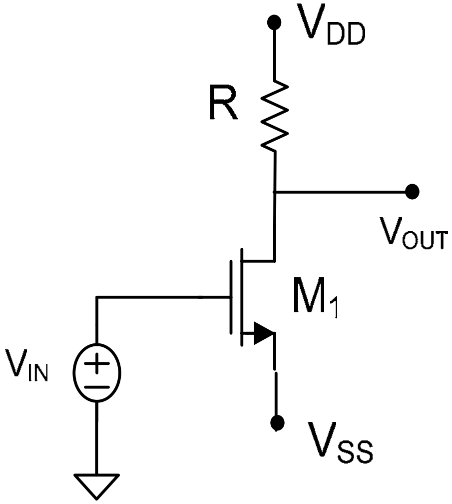
Operating point is often termed Q-point

Will be extended to functions of two and three variables

Small-signal Operation of Nonlinear Circuits

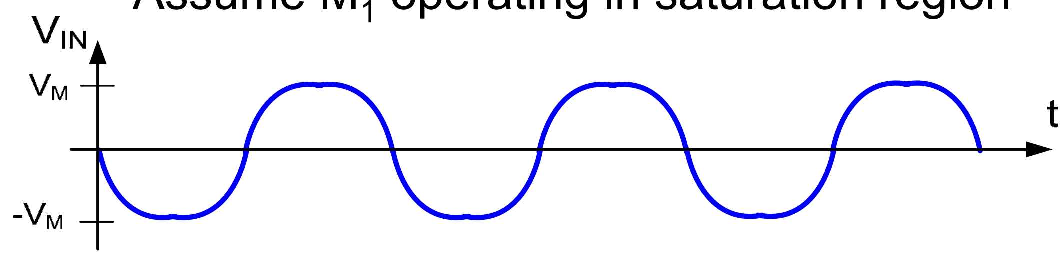
- Small-signal principles
- Example Circuit
- Small-Signal Models
- Small-Signal Analysis of Nonlinear Circuits

Small signal analysis example



By selecting appropriate value of V_{SS}, M_1 will operate in the saturation region

Assume M_1 operating in saturation region



$V_{IN}=V_M \sin\omega t$

V_M is small

$$V_{OUT}=V_{DD}-I_D R$$

$$I_D=\frac{\mu C_{OX} W}{2L}\left(V_{IN}-V_{SS}-V_T\right)^2$$

$$I_{DQ}=\frac{\mu C_{OX} W}{2L}\left(V_{SS}+V_T\right)^2$$

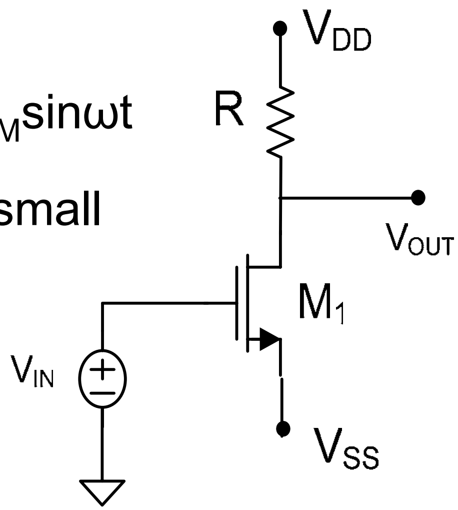
$$V_{OUT}=V_{DD}-\frac{\mu C_{OX} W}{2L}\left(V_{IN}-V_{SS}-V_T\right)^2 R$$

$$V_{OUT}=V_{DD}-\frac{\mu C_{OX} W}{2L}\left(V_M \sin\omega t-\left[V_{SS}+V_T\right]\right)^2 R$$

Small signal analysis example

$V_{IN}=V_M \sin\omega t$

V_M is small



$$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_M \sin\omega t-[V_{SS}+V_T]\right)^2 R$$

$$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2\left(1-\frac{V_M \sin\omega t}{[V_{SS}+V_T]}\right)^2 R$$

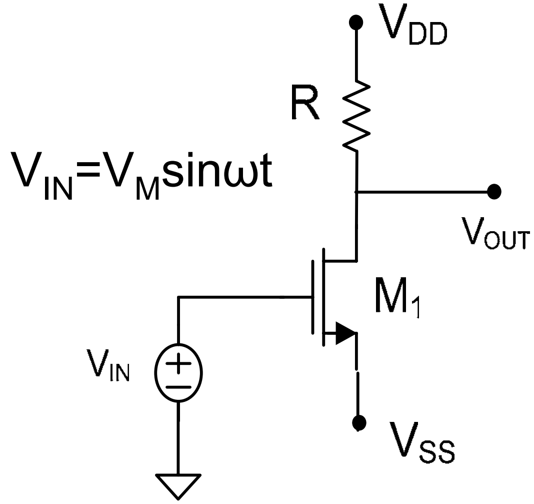
Recall that if x is small $(1+x)^2 \cong 1+2x$

$$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2\left(1-\frac{2V_M \sin\omega t}{[V_{SS}+V_T]}\right) R$$

$$V_{OUT}=\left\{V_{DD}-\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2 R\right\}+\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2\left(\frac{2V_M \sin\omega t}{[V_{SS}+V_T]}\right) R$$

$$V_{OUT}=\left\{V_{DD}-\frac{\mu C_{OX}W}{2L}[V_{SS}+V_T]^2 R\right\}+\left\{\frac{\mu C_{OX}W}{L}[V_{SS}+V_T] R\right\}V_M \sin\omega t$$

Small signal analysis example

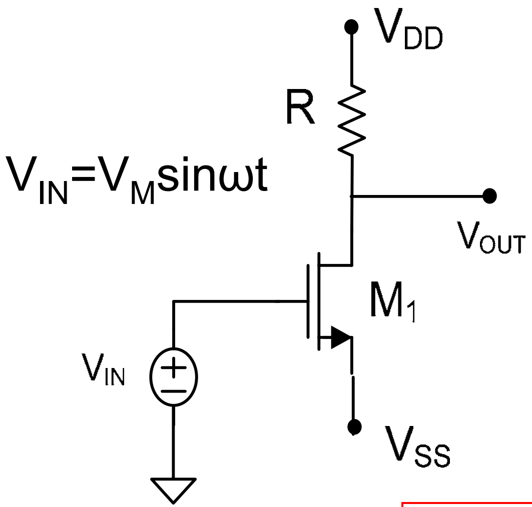


By selecting appropriate value of V_{SS}, M_1 will operate in the saturation region

Assume M_1 operating in saturation region

$$V_{OUT} = \left\{V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS} + V_T\right]^2 R\right\} + \left\{\frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R\right\} V_M \sin\omega t$$

Small signal analysis example



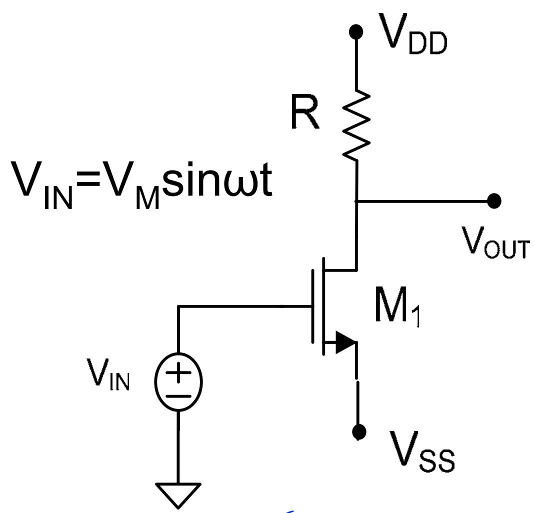
Assume M_1 operating in saturation region

$$V_{OUT} = \underbrace{\left\{V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS} + V_T\right]^2 R\right\}}_{\text{Quiescent Output}} + \underbrace{\left\{\frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right]R\right\}}_{\text{ss Voltage Gain}} V_M \sin\omega t$$

$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right]R$$

But – this expression gives little insight into how large the gain is !

Small signal analysis example



$$V_{OUT} = \left\{ V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS} + V_T\right]^2 R \right\} + \left\{ \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R \right\} V_M \sin\omega t$$

$$A_v = \frac{\mu C_{ox} W}{L}\left[V_{SS} + V_T\right] R$$

But recall:

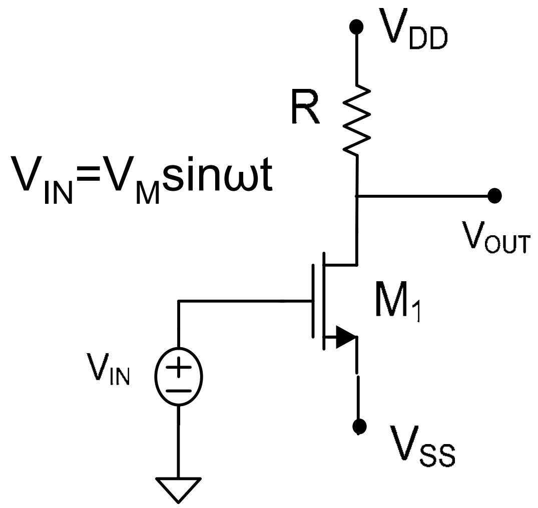
$$I_{DQ} = \frac{\mu C_{OX} W}{2L}\left(V_{SS} + V_T\right)^2$$

Thus, substituting from the expression for I_{DQ} we obtain

$$A_v = \frac{2 I_{DQ} R}{\left[V_{SS} + V_T\right]}$$

Note this is negative since $V_{SS}+V_T < 0$

Small signal analysis example



$$A_v = \frac{2I_{DQ}R}{[V_{SS}+V_T]}$$

Observe the small signal voltage gain is twice the Quiescent voltage across R divided by $V_{SS}+V_T$

- This analysis which required linearization of a nonlinear output voltage is quite tedious.
- This approach becomes unwieldy for even slightly more complicated circuits
- A much easier approach based upon the development of small signal models will provide the same results, provide more insight into both analysis and design, and result in a dramatic reduction in computational requirements

Small-signal Operation of Nonlinear Circuits

- Small-signal principles
- Example Circuit
- Small-Signal Models
- Small-Signal Analysis of Nonlinear Circuits

Solution for the example was based upon solving the nonlinear circuit for V_{OUT} and then linear zing the solution by doing a Taylor's series expansion

- *Solution of nonlinear equations very involved with two or more nonlinear devices*

- *Taylor's series linearization can get very tedious if multiple nonlinear devices are present*

Standard Approach to small-signal analysis of nonlinear networks

1. *Solve nonlinear network*
2. *Linearize solution*

Alternative Approach to small-signal analysis of nonlinear networks

1. *Linearize nonlinear devices*
2. *Replace all devices with small-signal equivalent*
3. *Solve linear small-signal network*

Alternative Approach to small-signal analysis of nonlinear networks

1. *Linearize nonlinear devices*
2. *Replace all devices with small-signal equivalent*
3. *Solve linear small-signal network*

- **Must only develop linearized model once for any nonlinear device**

  e.g. once for a MOSFET, once for a JFET, and once for a BJT

  Linearized model for nonlinear device termed "small-signal model"

  derivation of small-signal model for most nonlinear devices is less complicated than solving even one simple nonlinear circuit

- **Solution of linear network much easier than solution of nonlinear network**

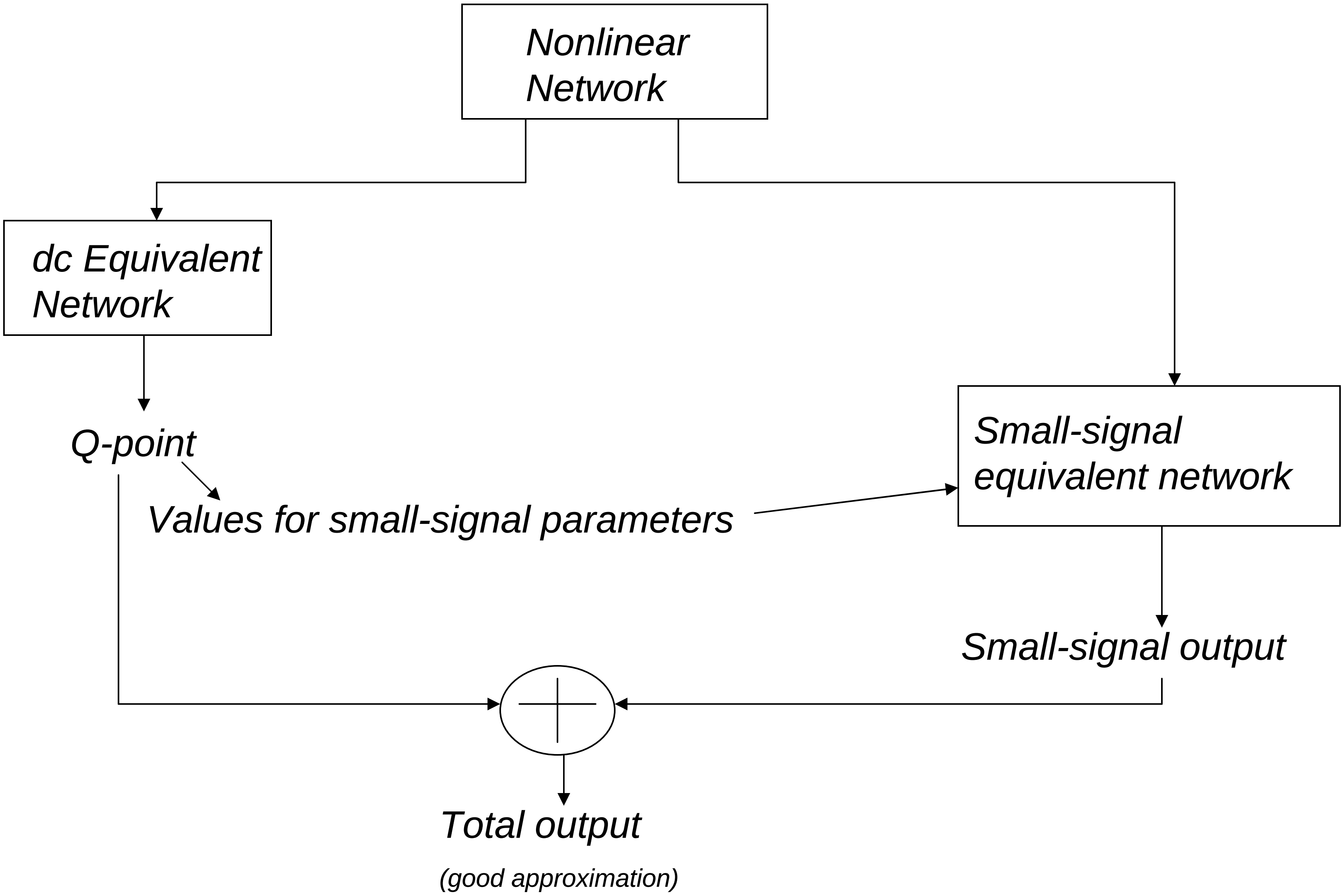
Standard Approach to small-signal analysis of nonlinear networks

1. *Linearize nonlinear devices*

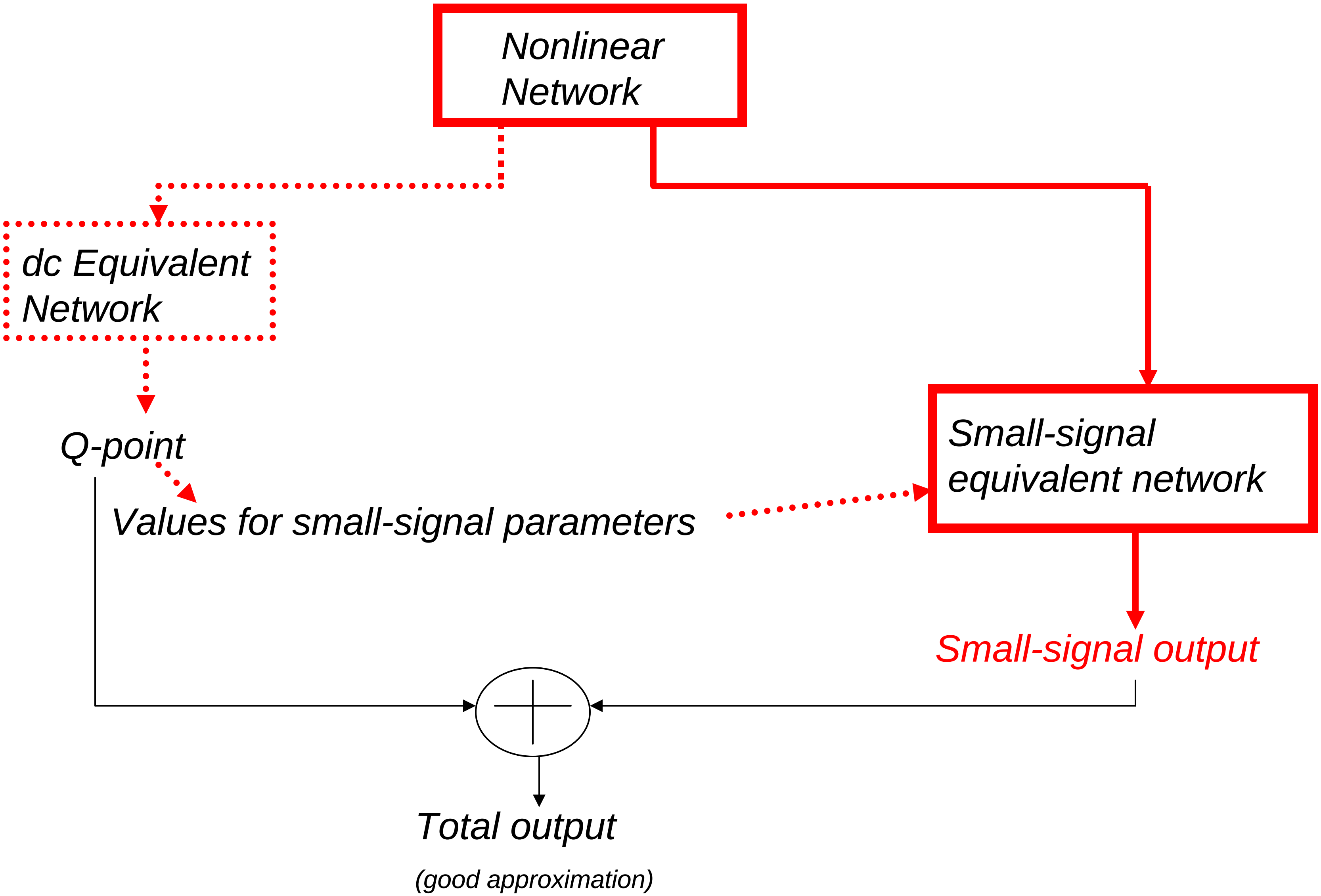
2. *Replace all devices with small-signal equivalent*

3. *Solve linear small-signal network*

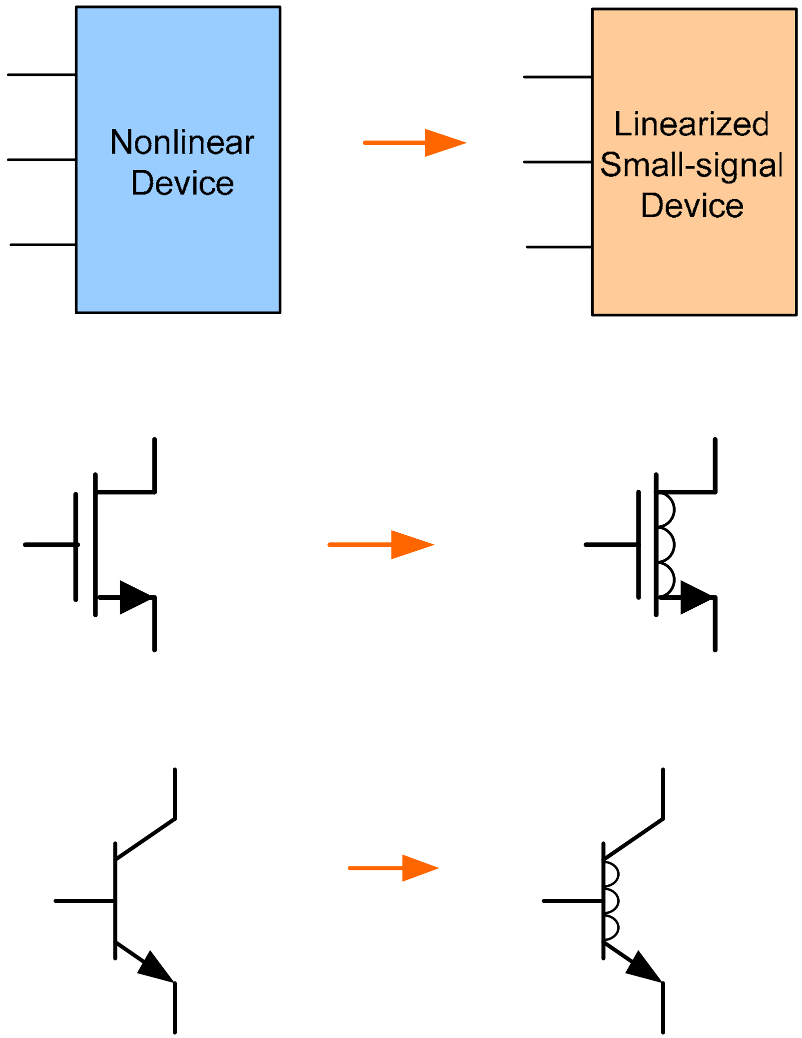
Standard Approach to analysis of nonlinear networks



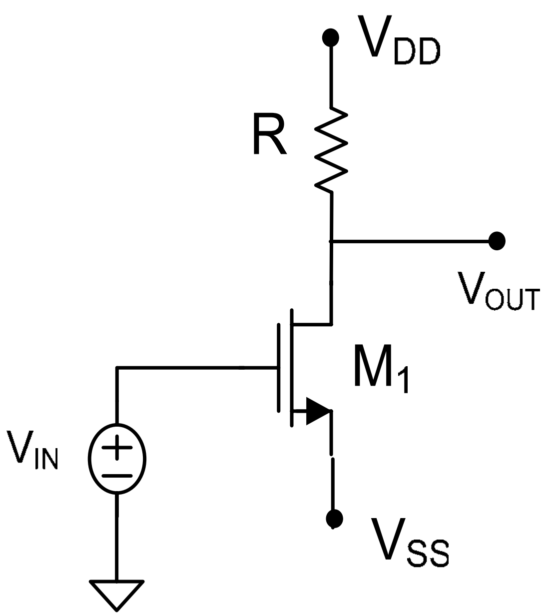
Standard Approach to small-signal analysis of nonlinear networks



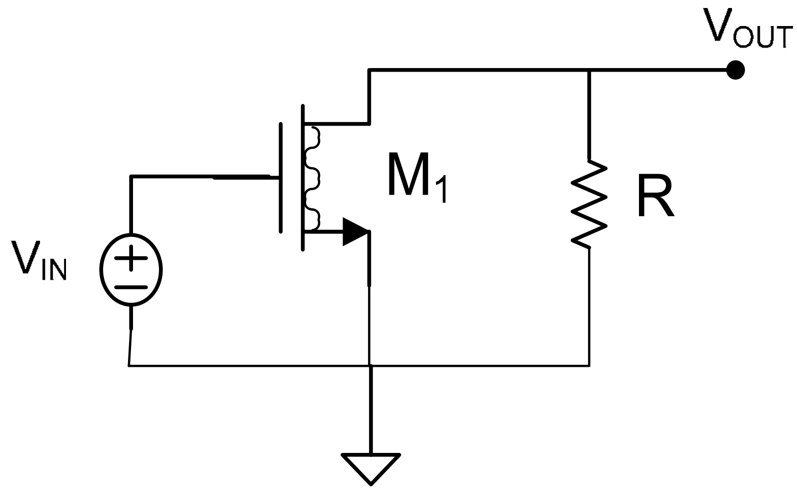
Linearized nonlinear devices



Example:

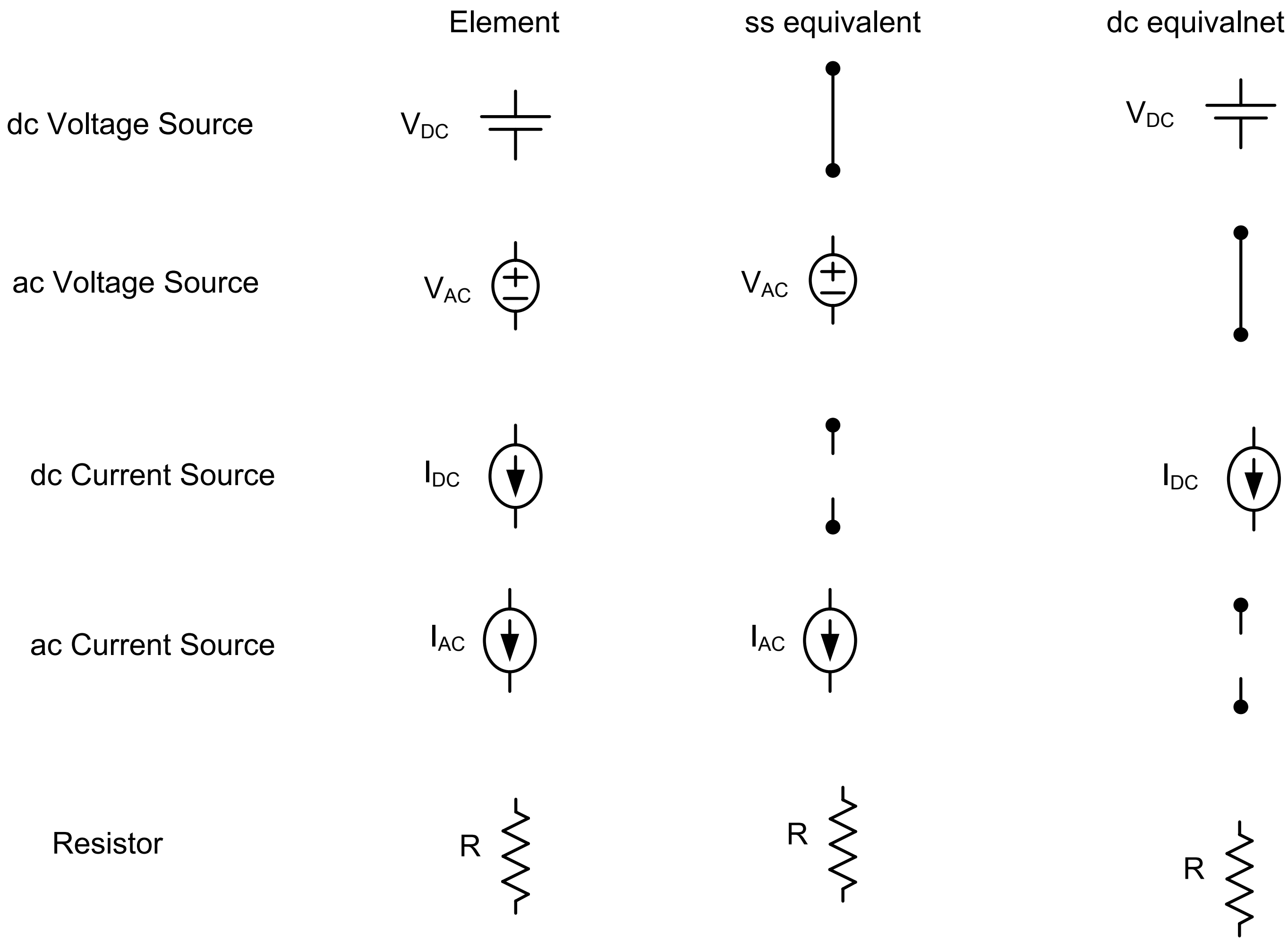


Nonlinear network

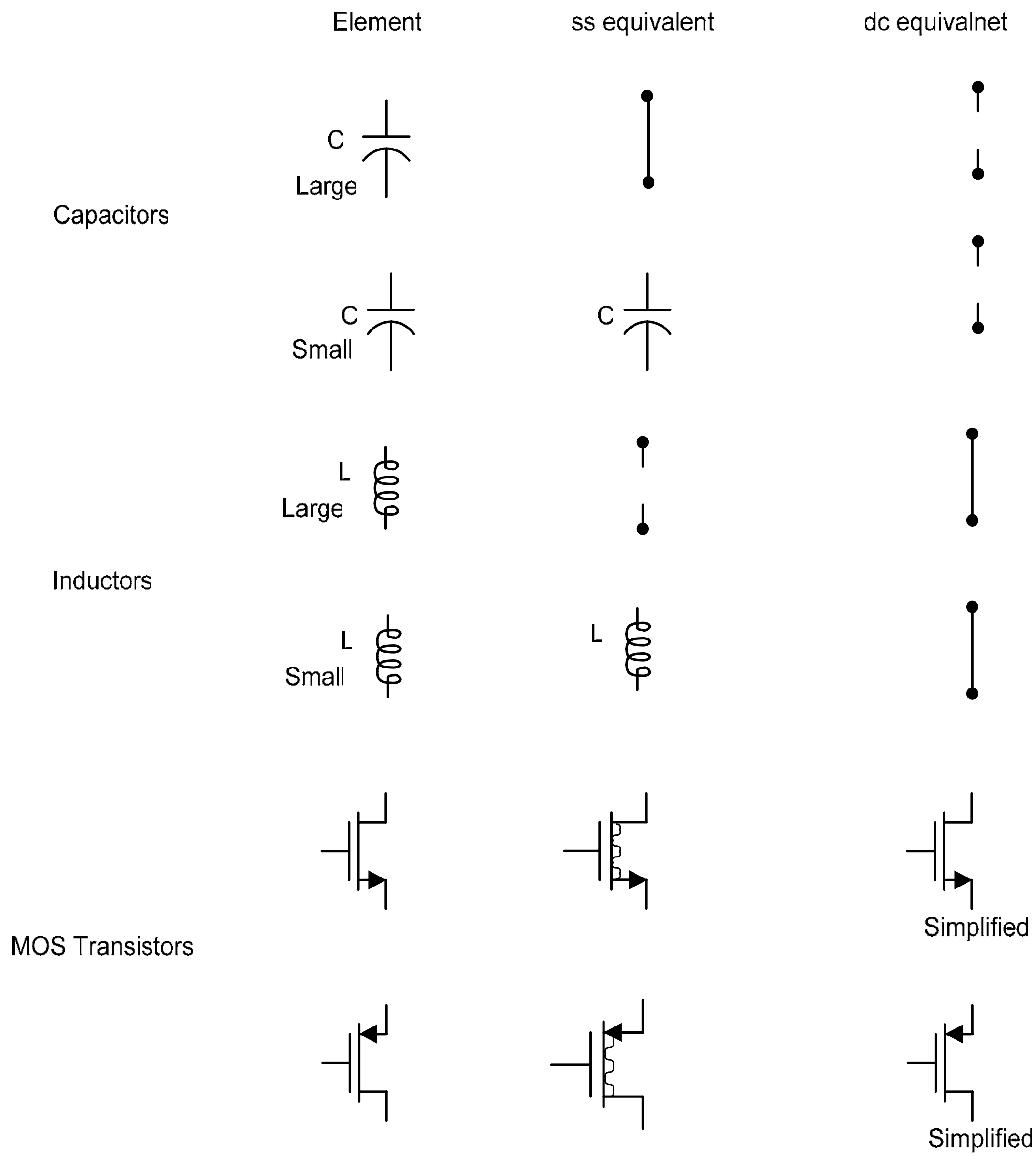


Linearized small-signal network

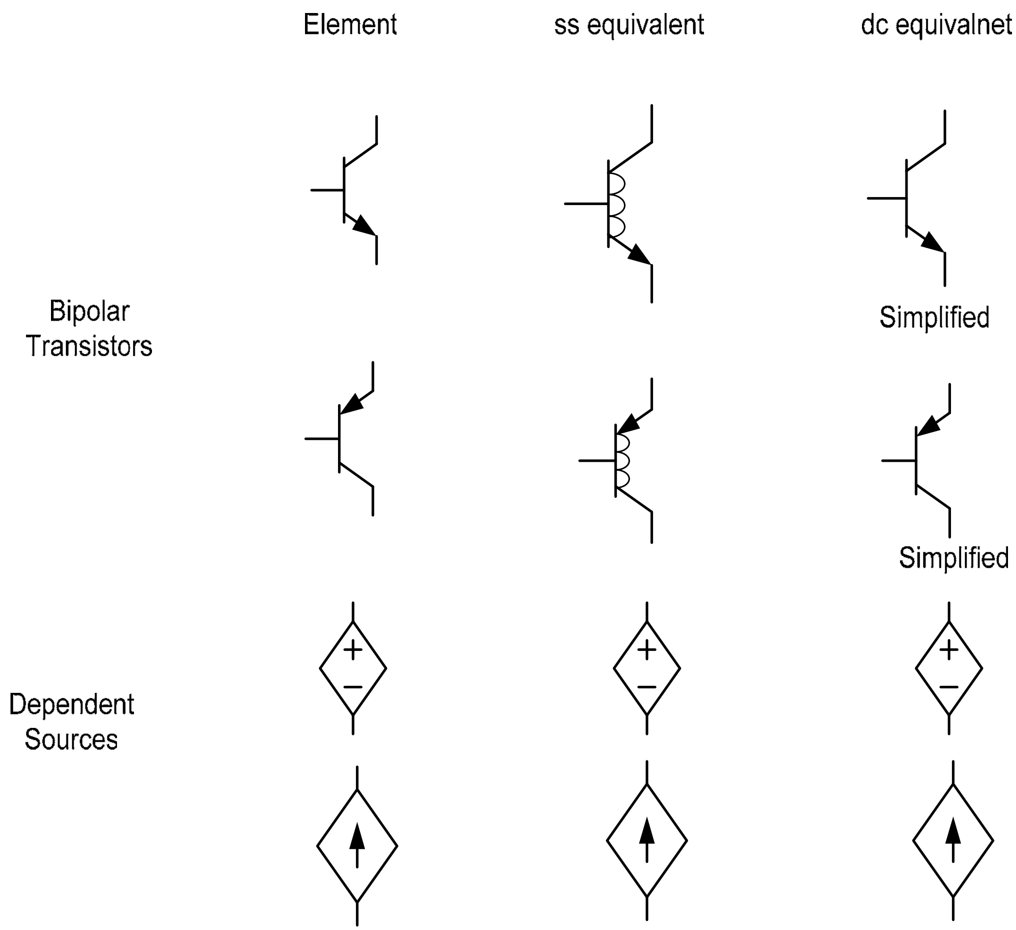
Dc and small-signal equivalent elements



Dc and small-signal equivalent elements

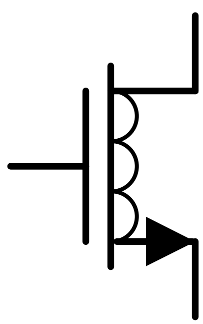


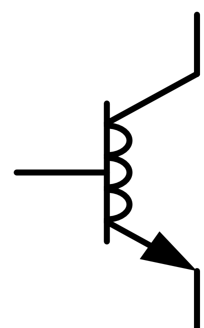
Dc and small-signal equivalent elements



How is the small-signal equivalent circuit obtained from the nonlinear circuit?

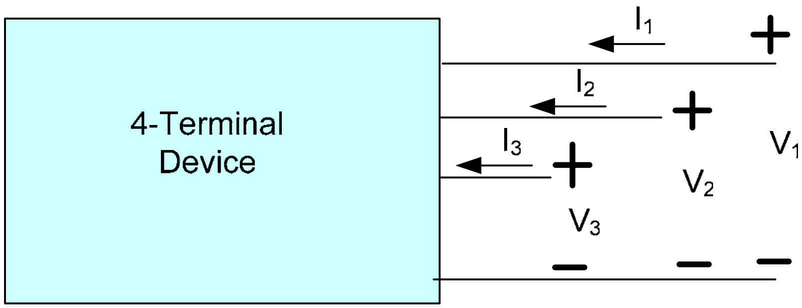
What is the small-signal equivalent of the MOSFET and BJT ?





Small-Signal Model

Consider 4-terminal network



$$\left.\begin{aligned} I_1 &= f_1(V_1, V_2, V_3) \\ I_2 &= f_2(V_1, V_2, V_3) \\ I_3 &= f_3(V_1, V_2, V_3) \end{aligned}\right\}$$

Define

$$i_1 = I_1 - I_{1Q} \qquad v_1 = V_1 - V_{1Q}$$

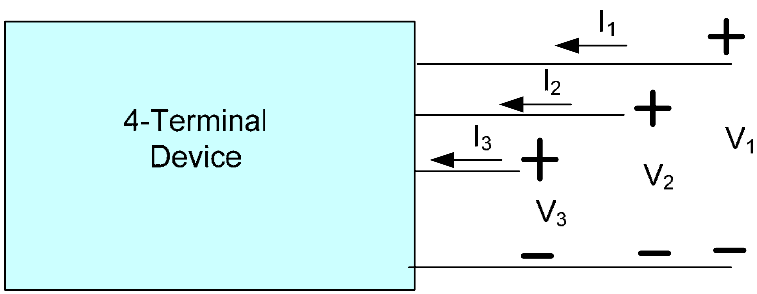
$$i_2 = I_2 - I_{2Q} \qquad v_2 = V_2 - V_{2Q}$$

$$i_3 = I_3 - I_{3Q} \qquad v_3 = V_3 - V_{3Q}$$

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents

Small-Signal Model

Consider 4-terminal network



$$i_1 = g_1(v_1, v_2, v_3)$$
$$i_2 = g_2(v_1, v_2, v_3)$$
$$i_1 = g_3(v_1, v_2, v_3)$$

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents

For small signals, this relationship should be linear

Can be thought of as a change in coordinate systems from the large signal coordinate system to the small-signal coordinate system

Recall for a function of one variable

$$y = f(x)$$

Taylor's Series Expansion about the point x_0

$$y = f(x) = f(x)\Big|_{x=x_0} + \frac{\partial f}{\partial x}\Big|_{x=x_0}(x - x_0) + \frac{\partial^2 f}{\partial x^2}\Big|_{x=x_0}\frac{1}{2!}(x - x_0)^2 + \ldots$$

If x-x_0 is small

$$y \cong f(x)\Big|_{x=x_0} + \frac{\partial f}{\partial x}\Big|_{x=x_0}(x - x_0)$$

$$y \cong y_0 + \frac{\partial f}{\partial x}\Big|_{x=x_0}(x - x_0)$$

Recall for a function of one variable

$$y = f(x)$$

If x-x_0 is small

$$y \cong y_0 + \left.\frac{\partial f}{\partial x}\right|_{x=x_0} (x - x_0)$$

$$y - y_0 = \left.\frac{\partial f}{\partial x}\right|_{x=x_0} (x - x_0)$$

If we define the small signal variables as

$$\boldsymbol{y} = y - y_0$$

$$\boldsymbol{x} = x - x_0$$

Recall for a function of one variable

$$y = f(x)$$

If x-x_0 is small

$$y - y_0 = \left.\frac{\partial f}{\partial x}\right|_{x=x_0} (x - x_0)$$

If we define the small signal variables as

$$\boldsymbol{y} = y - y_0$$

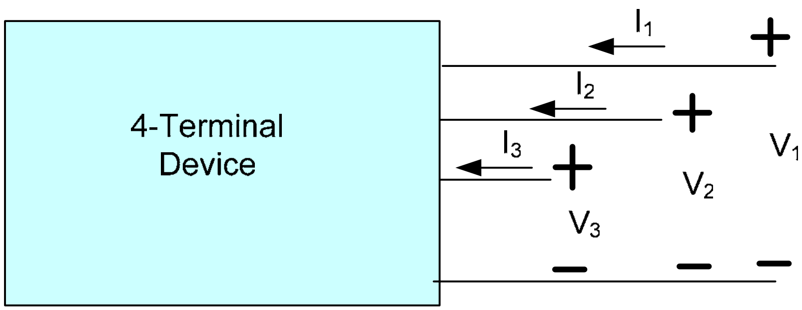
$$\boldsymbol{x} = x - x_0$$

Then

$$\boldsymbol{y} = \left.\frac{\partial f}{\partial x}\right|_{x=x_0} \boldsymbol{x}$$

This relationship is linear !

Consider 4-terminal network



$$\left.\begin{aligned} I_1 &= f_1(V_1, V_2, V_3) \\ I_2 &= f_2(V_1, V_2, V_3) \\ I_3 &= f_3(V_1, V_2, V_3) \end{aligned}\right\}$$

Nonlinear network characterized by 3 functions each functions of 3 variables

Consider now 3 functions each functions of 3 variables

$$\left.\begin{aligned} I_1 &= f_1(V_1, V_2, V_3) \\ I_2 &= f_2(V_1, V_2, V_3) \\ I_3 &= f_3(V_1, V_2, V_3) \end{aligned}\right\}$$

Define

$$\vec{V}_Q = \begin{bmatrix} V_{!Q} \\ V_{2Q} \\ V_{3Q} \end{bmatrix}$$

Consider now 3 functions each functions of 3 variables

$$\left.\begin{aligned} I_1 &= f_1(V_1, V_2, V_3) \\ I_2 &= f_2(V_1, V_2, V_3) \\ I_3 &= f_3(V_1, V_2, V_3) \end{aligned}\right\}$$

Define

$$\vec{V}_Q = \begin{bmatrix} V_{!Q} \\ V_{2Q} \\ V_{3Q} \end{bmatrix}$$

$$I_1 = f_1(V_1, V_2, V_3) \cong f_1(V_{1Q}, V_{2Q}, V_{3Q}) + \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1}\right|_{\vec{V}=\vec{V}_Q} (V_1 - V_{1Q}) + \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2}\right|_{\vec{V}=\vec{V}_Q} (V_2 - V_{2Q}) + \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3}\right|_{\vec{V}=\vec{V}_Q} (V_3 - V_{3Q})$$

$$I_1 - I_{1Q} = \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1}\right|_{\vec{V}=\vec{V}_Q} (V_1 - V_{1Q}) + \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2}\right|_{\vec{V}=\vec{V}_Q} (V_2 - V_{2Q}) + \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3}\right|_{\vec{V}=\vec{V}_Q} (V_3 - V_{3Q})$$

Consider now 3 functions each functions of 3 variables

$$I_1 - I_{1Q} = \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1}\right|_{\vec{V}=\vec{V}_Q}(V_1 - V_{1Q}) + \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2}\right|_{\vec{V}=\vec{V}_Q}(V_2 - V_{2Q}) + \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3}\right|_{\vec{V}=\vec{V}_Q}(V_3 - V_{3Q})$$

$$y_{11} = \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_1}\right|_{\vec{V}=\vec{V}_Q}$$

$$y_{12} = \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_2}\right|_{\vec{V}=\vec{V}_Q}$$

$$y_{13} = \left.\frac{\partial f_1(V_1, V_2, V_3)}{\partial V_3}\right|_{\vec{V}=\vec{V}_Q}$$

$$i_1 = I_1 - I_{1Q}$$

$$i_2 = I_2 - I_{2Q}$$

$$i_3 = I_3 - I_{3Q}$$

$$v_1 = V_1 - V_{1Q}$$

$$v_2 = V_2 - V_{2Q}$$

$$v_3 = V_3 - V_{3Q}$$

Consider now 3 functions each functions of 3 variables

$$\mathbf{I_1} - \mathbf{I_{1Q}} = \left.\frac{\partial \mathbf{f_1}(\mathbf{V_1},\mathbf{V_2},\mathbf{V_3})}{\partial \mathbf{V_1}}\right|_{\vec{\mathbf{V}}=\vec{\mathbf{V}}_\mathbf{Q}} (\mathbf{V_1} - \mathbf{V_{1Q}}) + \left.\frac{\partial \mathbf{f_1}(\mathbf{V_1},\mathbf{V_2},\mathbf{V_3})}{\partial \mathbf{V_2}}\right|_{\vec{\mathbf{V}}=\vec{\mathbf{V}}_\mathbf{Q}} (\mathbf{V_2} - \mathbf{V_{2Q}}) + \left.\frac{\partial \mathbf{f_1}(\mathbf{V_1},\mathbf{V_2},\mathbf{V_3})}{\partial \mathbf{V_3}}\right|_{\vec{\mathbf{V}}=\vec{\mathbf{V}}_\mathbf{Q}} (\mathbf{V_3} - \mathbf{V_{3Q}})$$

$$\boldsymbol{i}_1 = y_{11}\boldsymbol{v}_1 + y_{12}\boldsymbol{v}_2 + y_{13}\boldsymbol{v}_3$$

This is now a linear relationship between the small signal electrical variables

Consider now 3 functions each functions of 3 variables

$$i_1 = y_{11}v_1 + y_{12}v_2 + y_{13}v_3$$

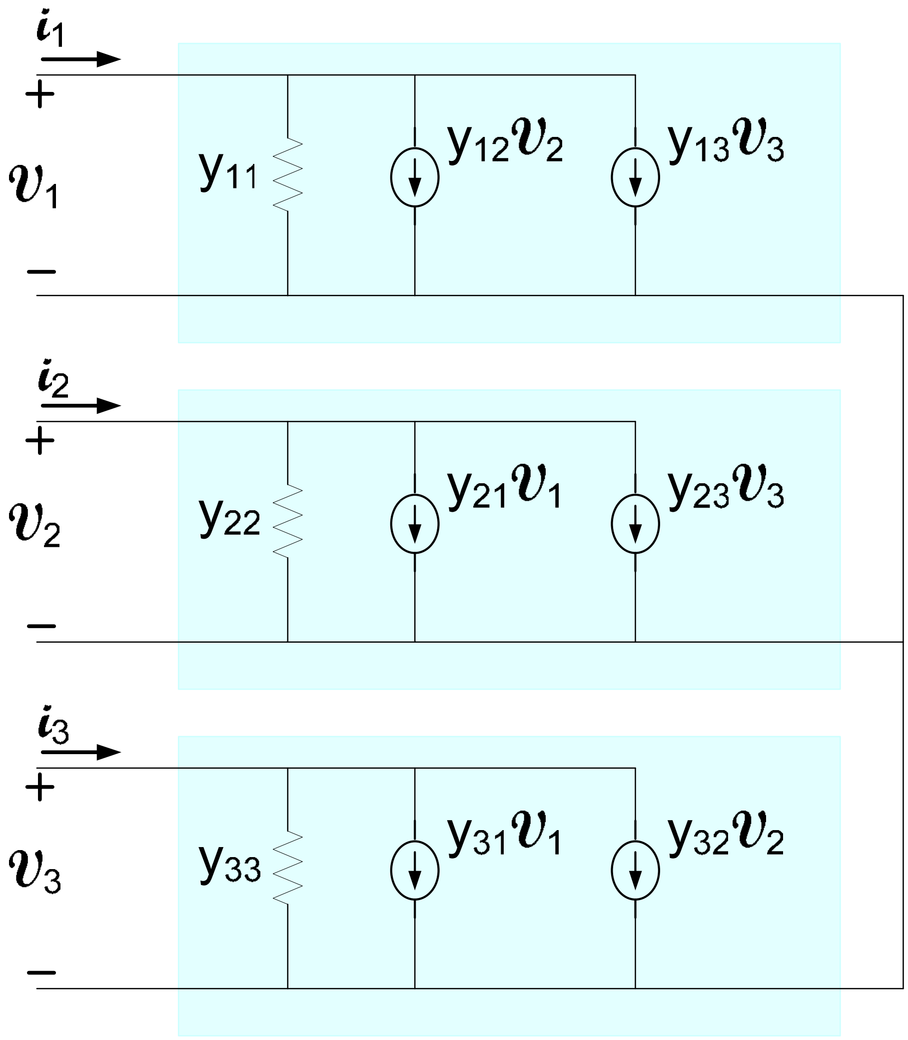
Lets now extend this to I_2 and I_3

Define $$y_{ij} = \left.\frac{\partial f_i(V_1, V_2, V_3)}{\partial V_j}\right|_{\vec{V}=\vec{V}_Q}$$

$$\boxed{\begin{aligned} i_1 &= y_{11}v_1 + y_{12}v_2 + y_{13}v_3 \\ i_2 &= y_{21}v_1 + y_{22}v_2 + y_{23}v_3 \\ i_3 &= y_{31}v_1 + y_{32}v_2 + y_{33}v_3 \end{aligned}}$$

This is a small-signal model of a 4-terminal network and it is linear
9 small-signal parameters characterize the linear 4-terminal network
Small-signal model parameters dependent upon Q-point !

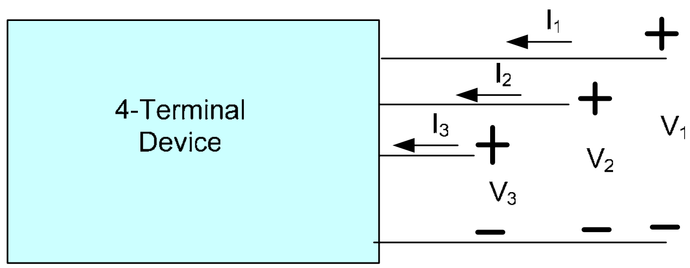
A small-signal equivalent circuit of a 4-terminal nonlinear network



$$y_{ij} = \left. \frac{\partial f_i(V_1, V_2, V_3)}{\partial V_j} \right|_{\vec{V}=\vec{V}_Q}$$

Equivalent circuit is not unique

4-terminal small-signal network summary

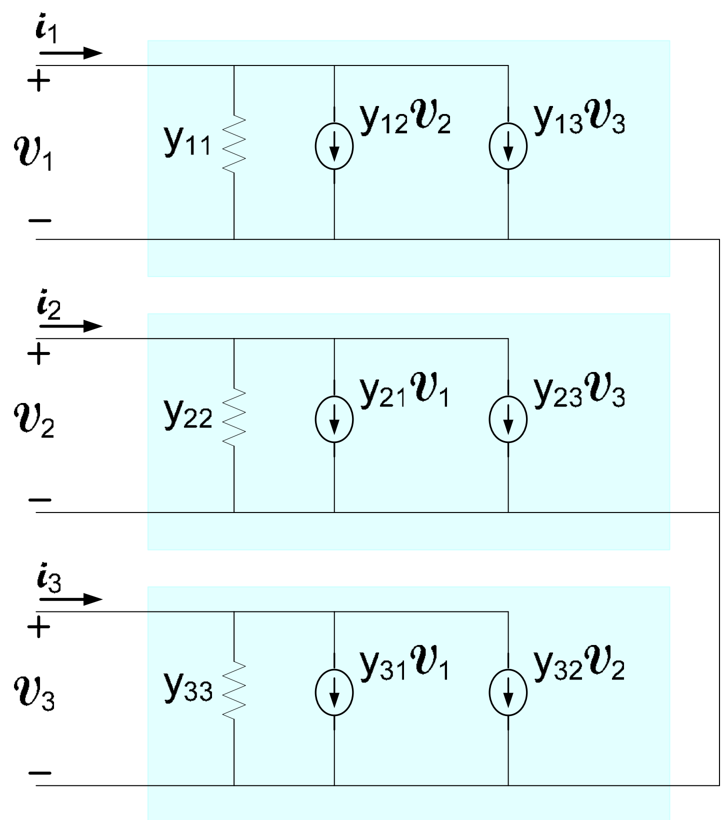


$$\left.\begin{aligned} I_1 &= f_1(V_1, V_2, V_3) \\ I_2 &= f_2(V_1, V_2, V_3) \\ I_3 &= f_3(V_1, V_2, V_3) \end{aligned}\right\}$$

Small signal model:

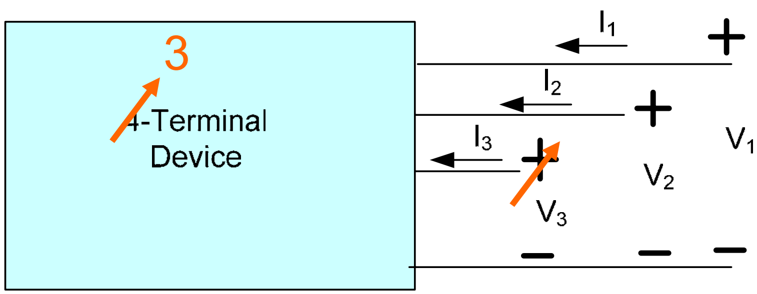
$$\boxed{\begin{aligned} i_1 &= y_{11} v_1 + y_{12} v_2 + y_{13} v_3 \\ i_2 &= y_{21} v_1 + y_{22} v_2 + y_{23} v_3 \\ i_3 &= y_{31} v_1 + y_{32} v_2 + y_{33} v_3 \end{aligned}}$$

$$y_{ij} = \left.\frac{\partial f_i(V_1, V_2, V_3)}{\partial V_j}\right|_{\vec{V} = \vec{V}_Q}$$



Consider 3-terminal network

Small-Signal Model



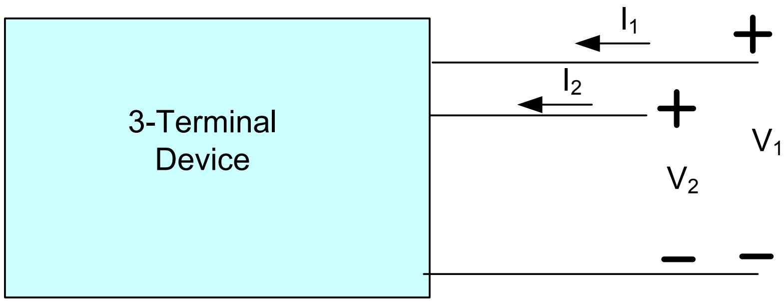
$$
\begin{aligned}
i_1 &= g_1(v_1, v_2, v_3) \\
i_2 &= g_2(v_1, v_2, v_3) \\
i_1 &= g_3(v_1, v_2, v_3)
\end{aligned}
$$

$$
\begin{aligned}
i_1 &= y_{11} v_1 + y_{12} v_2 + y_{13} v_3 \\
i_2 &= y_{21} v_1 + y_{22} v_2 + y_{23} v_3 \\
i_3 &= y_{31} v_1 + y_{32} v_2 + y_{33} v_3
\end{aligned}
$$

$$
y_{ij} = \left. \frac{\partial f_i(V_1, V_2, V_3)}{\partial V_j} \right|_{\vec{V} = \vec{V}_Q}
$$

Consider 3-terminal network

Small-Signal Model



$$\left.\begin{aligned} I_1 &= f_1(V_1, V_2) \\ I_2 &= f_2(V_1, V_2) \end{aligned}\right\}$$

Define

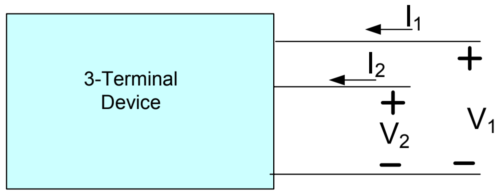
$$i_1 = I_1 - I_{1Q} \qquad v_1 = V_1 - V_{1Q}$$

$$i_2 = I_2 - I_{2Q} \qquad v_2 = V_2 - V_{2Q}$$

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents

Consider 3-terminal network

Small-Signal Model

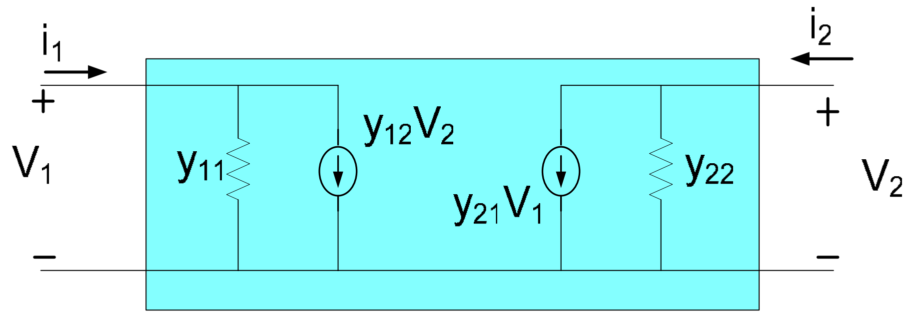


$$i_1 = y_{11}v_1 + y_{12}v_2$$
$$i_2 = y_{21}v_1 + y_{22}v_2$$

$$\mathbf{y_{ij}} = \left.\frac{\partial \mathbf{f_i}(\mathbf{V_1},\mathbf{V_2})}{\partial \mathbf{V_j}}\right|_{\vec{\mathbf{V}}=\vec{\mathbf{V}}_\mathbf{Q}}$$

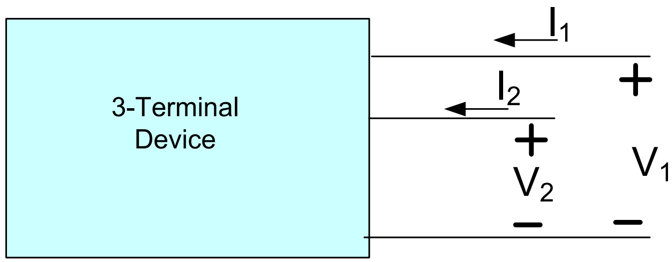
$$\vec{\mathbf{V}} = \begin{pmatrix} \mathbf{V_{1Q}} \\ \mathbf{V_{2Q}} \end{pmatrix}$$

A Small Signal Equivalent Circuit



4 small-signal parameters characterize this 3-terminal (two-port) linear network
Small signal parameters dependent upon Q-point

3-terminal small-signal network summary

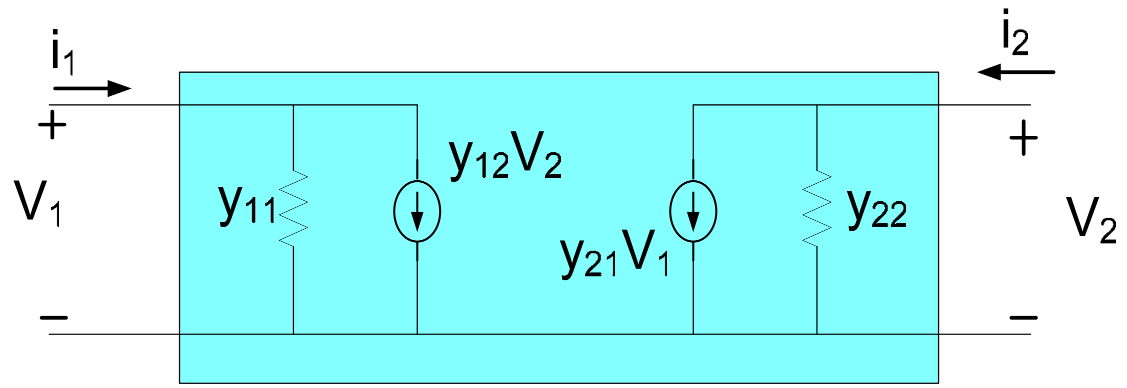


$$\left.\begin{aligned} \mathbf{I_1} &= \mathbf{f_1}(\mathbf{V_1},\mathbf{V_2}) \\ \mathbf{I_2} &= \mathbf{f_2}(\mathbf{V_1},\mathbf{V_2}) \end{aligned}\right\}$$

Small signal model:

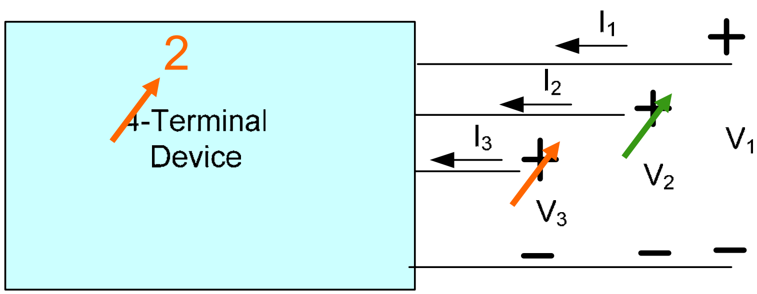
$$\boldsymbol{i}_1 = y_{11}\boldsymbol{v}_1 + y_{12}\boldsymbol{v}_2$$
$$\boldsymbol{i}_2 = y_{21}\boldsymbol{v}_1 + y_{22}\boldsymbol{v}_2$$

$$\mathbf{y_{ij}} = \left.\frac{\partial \mathbf{f_i}(\mathbf{V_1},\mathbf{V_2})}{\partial \mathbf{V_j}}\right|_{\vec{\mathbf{V}}=\vec{\mathbf{V}}_\mathbf{Q}}$$



Consider 2-terminal network

Small-Signal Model



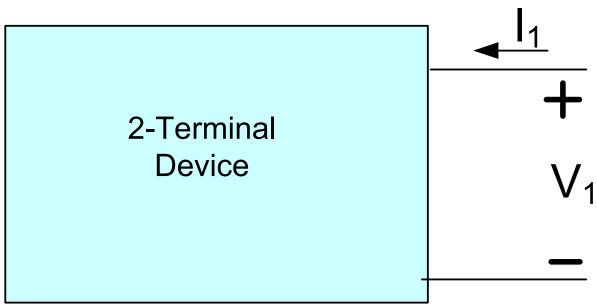
$$\left.\begin{aligned} i_1 &= g_1(v_1, v_2, v_3) \\ i_2 &= g_2(v_1, v_2, v_3) \\ i_1 &= g_3(v_1, v_2, v_3) \end{aligned}\right\}$$

$$\begin{aligned} i_1 &= y_{11} v_1 + y_{12} v_2 + y_{13} v_3 \\ i_2 &= y_{21} v_1 + y_{22} v_2 + y_{23} v_3 \\ i_3 &= y_{31} v_1 + y_{32} v_2 + y_{33} v_3 \end{aligned}$$

$$y_{ij} = \left.\frac{\partial f_i(V_1, V_2, V_3)}{\partial V_j}\right|_{\vec{V} = \vec{V}_Q}$$

Consider 2-terminal network

Small-Signal Model



$$I_1 = f_1(V_1)$$

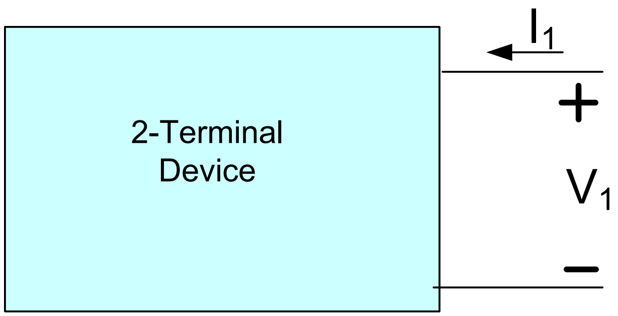
Define

$$\boldsymbol{i}_1 = I_1 - I_{1Q}$$
$$\boldsymbol{v}_1 = V_1 - V_{1Q}$$

Small signal model is that which represents the relationship between the small signal voltages and the small signal currents

Consider 2-terminal network

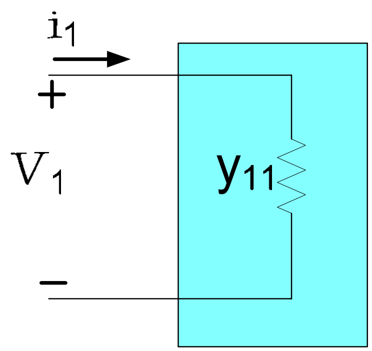
Small-Signal Model



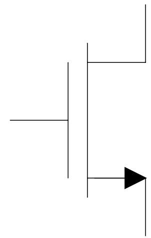
$$\boxed{i_1 = y_{11} v_1}$$

$$y_{11} = \left. \frac{\partial f_1(V_1)}{\partial V_1} \right|_{\vec{V} = \vec{V}_Q} \qquad \vec{V} = V_{1Q}$$

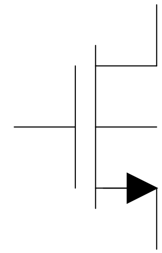
A Small Signal Equivalent Circuit



Small Signal Model of MOSFET



3-terminal device

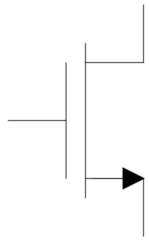


4-terminal device

MOSFET is actually a 4-terminal device but for many applications acceptable predictiions of performance can be obtained by treating it as a 3-terminal device by neglecting the bulk terminal

In this course, we have been treating it as a 3-terminal device and in this lecture will develop the small-signal model by treating it as a 3-terminal device

Small Signal Model of MOSFET

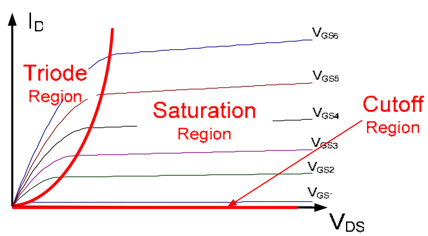


Large Signal Model

$$I_G = 0$$

3-terminal device

$$I_D = \begin{cases} 0 & V_{GS} \le V_T \\ \mu C_{OX} \dfrac{W}{L}\left(V_{GS} - V_T - \dfrac{V_{DS}}{2}\right)V_{DS} & V_{GS} \ge V_T \quad V_{DS} < V_{GS} - V_T \\ \mu C_{OX} \dfrac{W}{2L}\left(V_{GS} - V_T\right)^2\left(1 + \lambda V_{DS}\right) & V_{GS} \ge V_T \quad V_{DS} \ge V_{GS} - V_T \end{cases}$$



MOSFET is usually operated in saturation region in linear applications where a small-signal model is needed so will develop the small-signal model in the saturation region

Small Signal Model of MOSFET

$$I_1 = f_1(V_1, V_2) \quad \Longleftrightarrow \quad I_G = 0$$

$$I_2 = f_2(V_1, V_2) \quad \Longleftrightarrow \quad I_D = \mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

Small-signal model:

$$y_{ij} = \left. \frac{\partial f_i(V_1, V_2)}{\partial V_j} \right|_{\vec{V} = \vec{V}_Q}$$

$$y_{11} = \left. \frac{\partial I_G}{\partial V_{GS}} \right|_{\vec{V} = \vec{V}_Q} \qquad y_{12} = \left. \frac{\partial I_G}{\partial V_{DS}} \right|_{\vec{V} = \vec{V}_Q}$$

$$y_{21} = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{\vec{V} = \vec{V}_Q} \qquad y_{22} = \left. \frac{\partial I_D}{\partial V_{DS}} \right|_{\vec{V} = \vec{V}_Q}$$

Small Signal Model of MOSFET

$$I_1 = f_1(V_1, V_2) \Longleftrightarrow I_G = 0$$

$$I_2 = f_2(V_1, V_2) \Longleftrightarrow I_D = \mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^2 (1 + \lambda V_{DS})$$

Small-signal model:

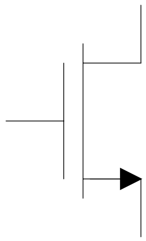
$$y_{11} = \left.\frac{\partial I_G}{\partial V_{GS}}\right|_{\vec{V}=\vec{V}_Q} = 0 \qquad y_{12} = \left.\frac{\partial I_G}{\partial V_{DS}}\right|_{\vec{V}=\vec{V}_Q} = 0$$

$$y_{21} = \left.\frac{\partial I_D}{\partial V_{GS}}\right|_{\vec{V}=\vec{V}_Q} = 2\mu C_{OX} \frac{W}{2L}(V_{GS} - V_T)^1 \left.(1 + \lambda V_{DS})\right|_{\vec{V}=\vec{V}_Q} = \mu C_{OX} \frac{W}{L}(V_{GSQ} - V_T)(1 + \lambda V_{DSQ})$$

$$y_{21} =\cong \mu C_{OX} \frac{W}{L}(V_{GSQ} - V_T)$$

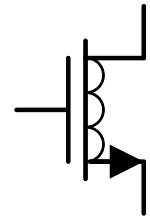
$$y_{22} = \left.\frac{\partial I_D}{\partial V_{DS}}\right|_{\vec{V}=\vec{V}_Q} = \mu C_{OX} \frac{W}{2L}\left.(V_{GS} - V_T)^2 \lambda\right|_{\vec{V}=\vec{V}_Q} \cong \lambda I_{DQ}$$

Small Signal Model of MOSFET



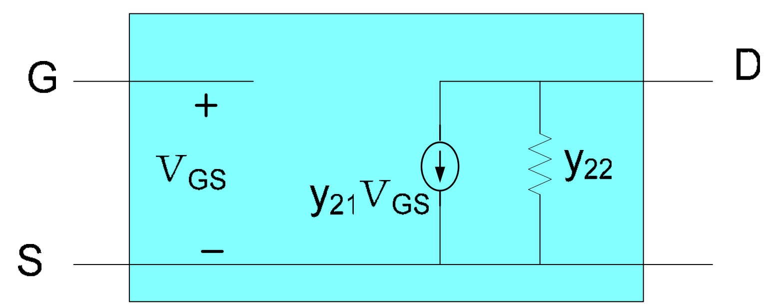
$$I_G = 0$$

$$I_D = \mu C_{OX} \frac{W}{2L}\left(V_{GS} - V_T\right)^2\left(1 + \lambda V_{DS}\right)$$

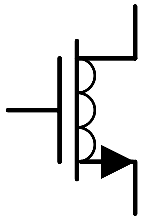


$$y_{12} = 0$$
$$y_{11} = 0$$
$$y_{21} \cong \mu C_{OX} \frac{W}{L}\left(V_{GSQ} - V_T\right)$$
$$y_{22} \cong \lambda I_{DQ}$$

$$i_G = y_{11} v_{GS} + y_{12} v_{DS}$$
$$i_D = y_{21} v_{GS} + y_{22} v_{DS}$$



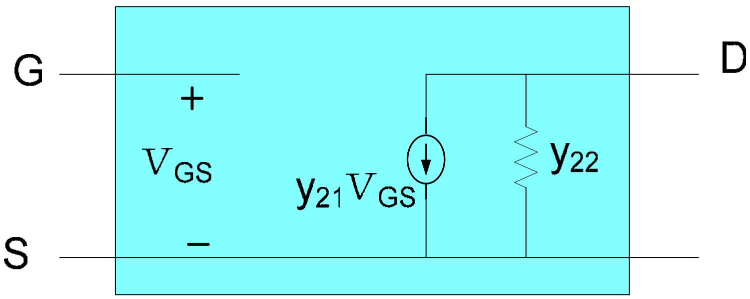
Small Signal Model of MOSFET

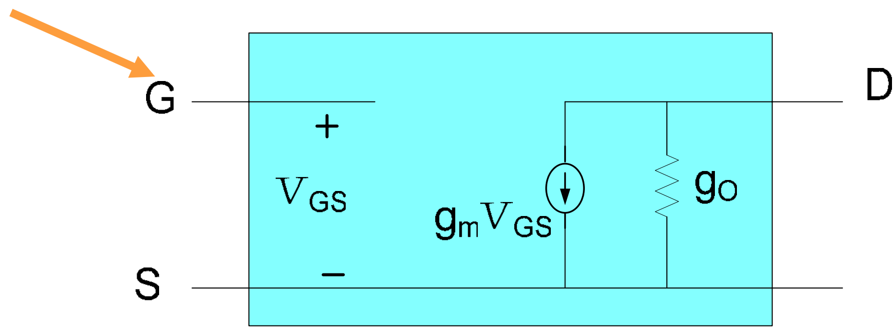


by convention, $y_{21}=g_m$, $y_{22}=g_0$

$$\therefore \quad y_{21} \cong g_m = \mu C_{OX} \frac{W}{L}(V_{GSQ} - V_T)$$

$$y_{22} = g_o \cong \lambda I_{DQ}$$

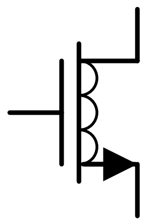




$$i_G = 0$$

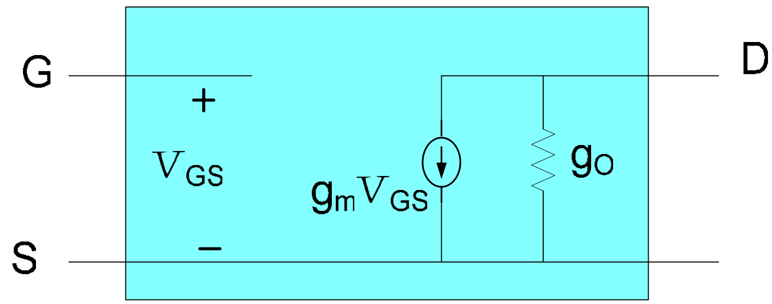
$$i_D = g_m v_{GS} + g_o v_{DS}$$

Small Signal Model of MOSFET



$$g_m = \mu C_{OX} \frac{W}{L}\left(V_{GSQ} - V_T\right)$$

$$g_o \cong \lambda I_{DQ}$$



Alternate equivalent expressions:

$$I_{DQ} = \mu C_{OX} \frac{W}{2L}\left(V_{GSQ} - V_T\right)^2 \left(1 + \lambda V_{DSQ}\right) \cong \mu C_{OX} \frac{W}{2L}\left(V_{GSQ} - V_T\right)^2$$

$$g_m = \mu C_{OX} \frac{W}{L}\left(V_{GSQ} - V_T\right)$$

$$g_m = \sqrt{2\mu C_{OX} \frac{W}{L}} \bullet \sqrt{I_{DQ}}$$

$$g_m = \frac{2I_{DQ}}{V_{GSQ} - V_T}$$